Sympathetic Vibratory Physics

Propositions of Proportion

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"My system, in every part and detail, both in the developing of this power and in every branch of its utilization, is based and founded on *sympathetic vibration*. In no other way would it be possible to awaken or develop this force, and equally impossible would it be to operate my engine upon any other principle."

John Keely, 1888

Theory of Proportion

"A proportion is an expression of equality between two equal ratios; and is written in one of the following forms:

$$a:b = c:d; \; a:b::c:d; \; a/b=c/d.$$

This proportion is read, "*a* is to *b* as *c* is to *d*"; or "the ratio of *a* to *b* is equal to the ratio of *c* to *d*."

The terms of a proportion are the four quantities compared; the first and third terms are called the *antecedents*, the second and fourth terms, the *consequents*; the first and fourth terms, the *extremes*, the second and third terms, the *means*.

Thus, in the proportion $a:b = c:d$; *a* and *c* are antecedents, *b* and *d* the consequents, *a* and *d* the extremes, *b* and *c* the means.

The fourth proportional to three given quantities is the fourth term of the proportion which has for its first three terms the three given quantities taken in order.

Thus, *d* is the fourth proportional to *a*, *b*, and *c* in the proportion

$$a:b = c:d.$$

The quantities *a, b, c, d, e,* are said to be in *continued* proportion, if $a:b + b:c = c:d = d:e$.

If three quantities are in *continued* proportion, the second is called the *mean proportional* between the other two, and the third is called the *third proportional* to the other two.

Thus, in the proportion $a:b = b:c$; *b* is the mean proportional between *a* and *c*; and *c* is the third proportional to *a* and *b*."

Propositions of Proportion

Proposition I

In every proportion the product of the extremes is equal to the product of the means.

Proposition II

The mean proportional between two quantities is equal to the square root of their products.

Proposition III

If the product of two quantities is equal to the product of two others, either two may be made the extremes of the proportion in which the other two are made the means.

Proposition IV

If four quantities are in proportion, they are in proportion by *alternation*; that is, the first term is to the third as the second is to the fourth.

Proposition V

If four quantities are in proportion, they are in proportion by *inversion*; that is, the second term is to the first as the fourth is to the third.

Proposition VI

If four quantities are in proportion, they are in proportion by *composition*; that is, the sum of the first two terms is to the second term as the sum of the last two terms is to the fourth term.

Proposition VII

If four quantities are in proportion, they are in proportion by *division*; that is, the difference of the first two terms is to the second term as the difference of the last two terms is to the fourth term.

Proposition VIII

If four quantities are in proportion, they are in proportion by *composition* and *division*; that is, the sum of the first two terms is to their difference as the sum of the last two terms is to their difference.

Proposition IX

In a series of equal ratios, the sum of the antecedents is to the sum of the consequents as any antecedent is to is consequent.

Proposition X

The products of the corresponding terms of two or more proportions are in proportion.

Corollary: If three quantities are in continued proportion, the first is to the third as the square of the first is to the square of the second.

Proposition XI

Like powers of the terms of a proportion are in proportion.

Proposition XII

Equimultiples of two quantities are in the same ratio as the quantities themselves.
[Equimultiples of two quantities are the products obtained by multiplying each of them by the same number. Thus, ma and mb are equimultiples of a and b.]

Proposition XIII

If a line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally.

Corollary 1: One side of a triangle is to either part cut off by a straight line parallel to the base as the other side is to the corresponding part.

Corollary 2: If two lines are cut by any number of parallels, the corresponding intercepts are proportional.

Proposition XIV

If a straight line divides two sides of a triangle proportionally, it is parallel to the third side.

Proposition XV

The bisector of an angle of a triangle divides the opposite side into segments which are proportional to the adjacent sides.

Proposition XVI

The bisector of an exterior angle of a triangle divides the opposite side externally into segments which are proportional to the adjacent sides.

Corollary: The bisectors of an interior angle and an exterior angle at one vertex of a triangle meeting the opposite side divide that side *harmonically*.
[If a given straight line is divided internally and externally into segments having the same ratio, the line is said to be *divided harmonically*.]

Proposition XVII

Two mutually equiangular triangles are similar.

Corollary 1: Two triangles are similar if two angles of the one are equal, respectively, to two angles of the other.

Corollary 2: Two right triangles are similar if an acute angle of the one is equal to an acute angle of the other.

Proposition XVIII

If two triangles have an angle of the one equal to an angle of the other, and the including sides proportional, they are similar.

Proposition XIX

If two triangles have their sides respectively proportional, they are similar.

Proposition XX

Two triangles which have their sides respectively parallel, or respectively perpendicular, are similar.

Corollary: The parallel sides and the perpendicular sides are *homologous* sides of the triangle.

Proposition XXI

The homologous altitudes of two similar triangles have the same ratio as any two homologous sides.

Proposition XXII

If two parallels are cut by three or more transversals that pass through the same point, the corresponding segments are proportional.

Proposition XXIII

Conversely: If three or more non-parallel straight lines intercept proportional segments upon two parallels, they pass through a common point.

Proposition XXIV

The perimeters of two similar polygons have the same ratio as any two homologous sides.

Proposition XXV

If two polygons are similar, they are composed of the same number of triangles, similar each to each, and similarly placed.

Proposition XXVI

Conversely: If two polygons are composed of the same number of triangles, similar each to each, and similarly placed, the polygons are similar.

Source: *Plane and Solid Geometry*; Book III; pages 135-158. George A. Wentworth, 1902. Ginn & Company, The Athenæum Press, Boston, Massachusetts..